CSC-RUB PhD Project Proposal

Title: Fluctuations around Instanton solutions of the Navier-Stokes equation

Sector of research: Dr. rer. nat.

Degree awarded: Theoretical Physics

Keywords: turbulence and intermittency, instantons, optimal control, Gel'fand-Yaglom theory

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Research focus of supervisor: The main interests of TP I are, on the one side, the development of multiphysics/multiscale simulations of collisionless plasmas and, on the other side, the understanding of intermittency in turbulent fluids and plasmas using non-perturbative methods. The latter is the focus of this dissertation. In the last 10 years, great progress has been made by applying the instanton calculus to turbulence, which consists of five steps: (i) formulation of the turbulence problem as a path integral over all spatiotemporal forcing fluctuations (Onsager/Machlup or Janssen/de Dominicis), (ii) calculation of the saddle point solution (instanton) of the underlying path integral, (iii) calculation of the continuous symmetries and the corresponding contribution of the zero modes to the path integral, (iv) contribution of the fluctuations around the intantons, (v) contributions of the instanton gas. Our group at TP I, together with our international partners from Warwick, Rome and New York, has made important contributions to i), ii) and iv).

Publications:

1. T. Schorlepp, T. Grafke, R. Grauer, J. Phys. A: Math. Theor. **54** (2021) 235003
2. L. Ebener, G. Margazoglou, J.Friedrich, L. Biferale, R. Grauer, Chaos **29** (2019) 63102
3. T. Grafke, R. Grauer, T. Schäfer, E. Vanden-Eijnden, EPL **109** (2015) 34003
4. T. Grafke, R. Grauer, St. Schindel, Commun. Comput. Phys. **18** (2015) 577
5. T. Grafke, R. Grauer, T. Schäfer, J. Phys. A: Math. Theor. (Topical Review) **48** (2015) 333001

Summary of research plan

Background: Turbulence is still one of the last unsolved problems of classical physics. But what is the problem? Velocity fluctuations at different distances show a non-self-similar behaviour: Fluctuations measured at large distances follow a Gaussian distribution, while at small distances heavy tailed distributions occur. This phenomenon is called intermittency, and the unsolved problem is to compute this effect directly from the underlying Navier-Stokes equations. Various ingenious methods have been tried: quasi-normal approximation, Markovian approximations, direct interaction approximation (DIA), and renormalization group methods. The current overall understanding demands for non-perturbative methods: Fusion rules, functional renormalization, and instantons. The starting point is the path integral formulation of the stochastic Navier-Stokes system. The main challenge is to develop methods to approximate this integral. The instanton calculus is described above is the most prominent non-perturbative method for approximating the path integral. We succeeded in applying this technique to the simpler case of 1D Burgers turbulence (see Ref. 1). Already the calculation of instantons poses a major challenge in the 3D Navier-Stokes case. It leads to an optimization problem of size (spatial resolution)^3 x temporal resolution. In a typical simulation, this amounts to 1024^3x$2048 = 2.2\ 10^{12}$ degrees of freedom. The next step in the instanton calculus is the calculation of fluctuations around instantons. Gel'fand/Yaglom-like methods yield a matrix Riccati equation around instantons, as demonstrated in Ref. 1 and applied there to Burgers turbulence. The size of the matrix evolved by the Ricatti equations equals (spatial resolution)^6. Here, new techniques, e.g., low-rank tensor networks for matrix Ricatti equations, have to be devised.

Study objective: The primary objective is to develop new analytical and numerical methods for computing instantons and fluctuations around instantons for 3D Navier-Stokes turbulence. Among the new techniques for computing instantons is the development of an efficient collective coordinate approach as model reduction. The design of new approaches to solve the high-dimensional Ricatti equation will be based on tensor network (TN) techniques. TNs were introduced in many-particle physics allowing to escape the curse of dimensionality.
These techniques have demonstrated strong potential in various disciplines ranging from quantum materials to machine learning. Very promising results for handling Ricatti and Lyapunov
equations were presented by Breiten, Dolgov, Stoll, Numerical Algebra, Control & Optimization, **11** (2021) 407-429.

Furthermore, Ricatti equations can be linearized by a Radon transformation. This leads to a linear system that allows effective paralization over the initial conditions. However, the resulting equations require a careful stabilization strategy to handle fourth order dissipation.

Expected Results: The aim of this work is to obtain a more precise description of the probability distribution of turbulent velocity fluctuations on the basis of the non-perturbative instanton calculus. The approach is oriented from simpler descriptions of turbulence such as the one-dimensional Burgers equation, the two-dimensional GGS equation (Grafke-Grauer-Sideris) up to the three-dimensional Navier-Stokes equations. Publications are planned at each of these levels of complexity.

Methods: Both analytical and numerical methods are used. Analytical methods include stochastic systems, large deviation theory, path integral formulation of stochastic systems, instantons, optimal control and adjoint states. Numerical methods include pseudospectral simulations and numerical optimisation. A cluster of GPUs is available at the institute. Production runs are performed at the HPC centre of the FZ Jülich.

Candidate Requirements: MSc degree in Theoretical Physics or Applied Mathematics, basic knowledge of stochastic systems and tools of QFT; experience with programming languages and coding; sound knowledge of English.

Motivation for CSC application: You will work on one of the most challenging problems in classical physics, using innovative non-perturbative methods. Your work will be both analytical and numerical. We offer an exceptional research environment with integration into an international research community with locations in Warwick, UK (T. Grafke), Rome, Italy (L. Biferale) and New York, USA (T. Schäfer, E. Vanden-Eijnden). Here, I would like to emphasize at this point that my role as mentor of young scientists is reflected in the fact that 11 of my former students are now professors or in comparable research positions. In addition, locally you will be integrated in the Ruhr University Research School, for interdisciplinary skills development.